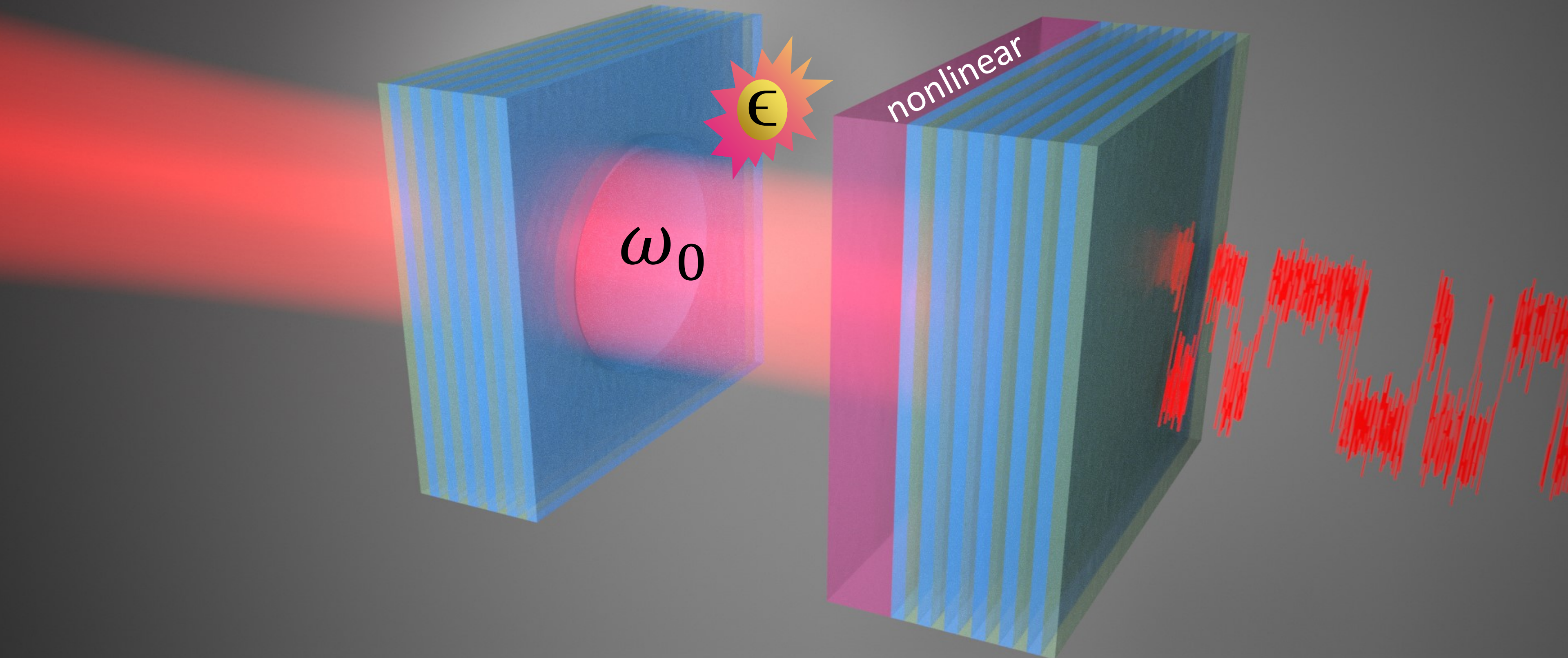
Noise-assisted sensing

Said R. K. Rodriguez
Dutch Photonics event
September 10, 2019

A bistable cavity as a sensor

- Detection speed and sensitivity enhanced by (quantum) fluctuations



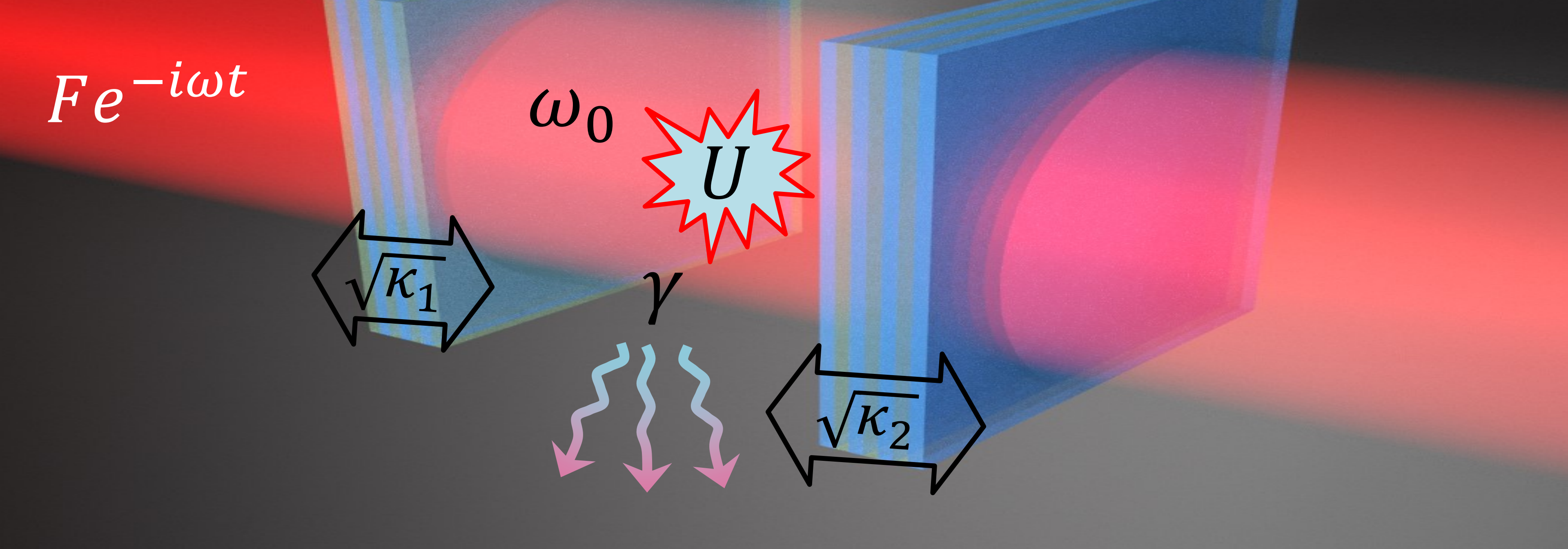
A classical nonlinear cavity

AMOLF physics of functional complex matter

$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \;\; - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F$$

detuning nonlinearity loss driving

$\Delta = \omega\text{-}\omega_0$ $\Gamma = \kappa_1 + \kappa_2 + \gamma$



A classical nonlinear cavity

AMOLF physics of functional complex matter

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detuning · nonlinearity · loss · driving

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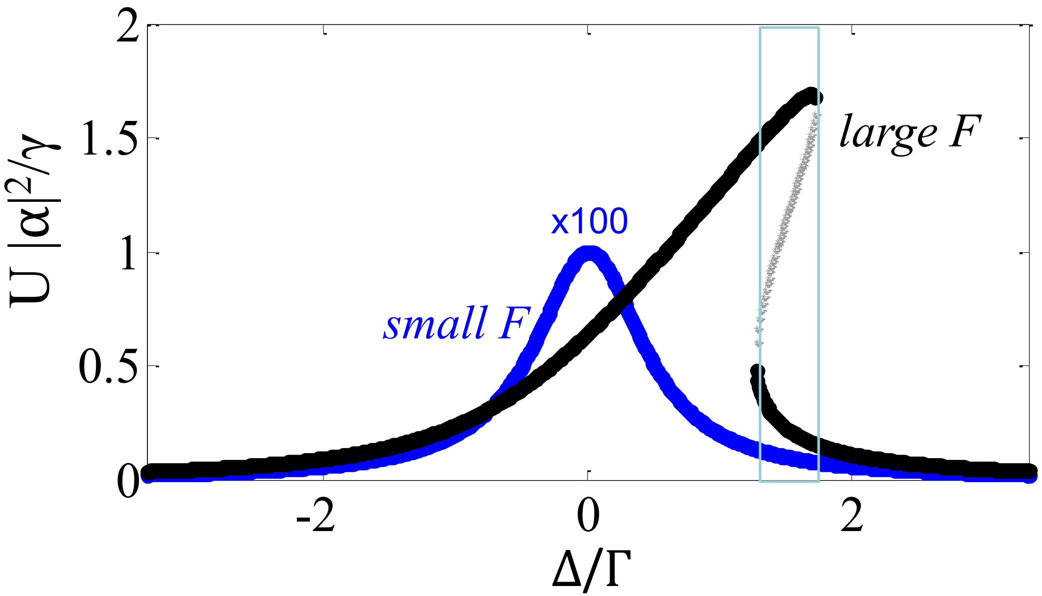
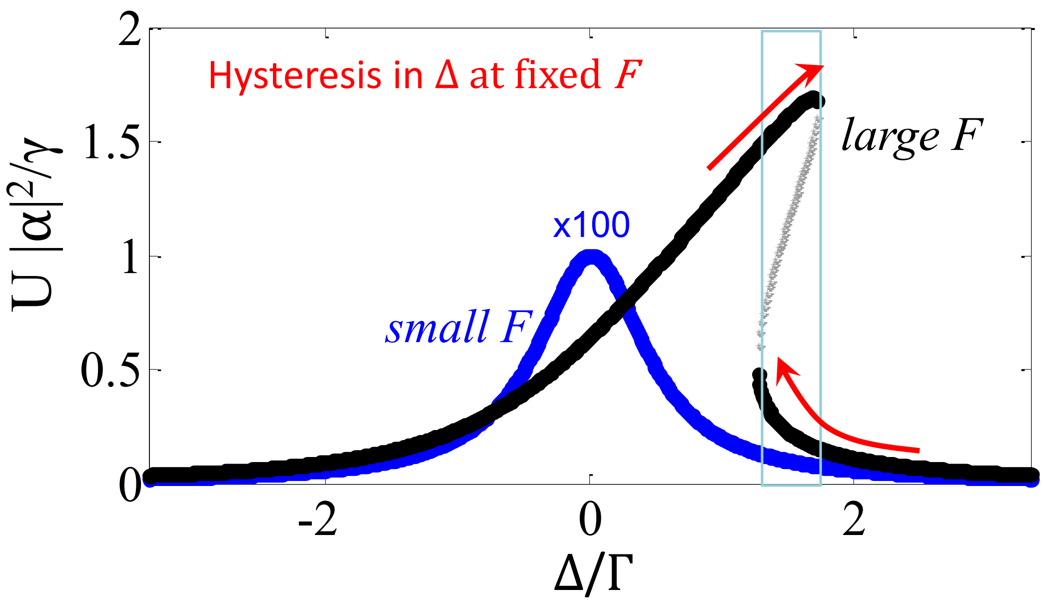


A classical nonlinear cavity

$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \;\; - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F$$

detuning (→ $-\Delta$), nonlinearity (→ U), loss (→ $\frac{\Gamma}{2}$), driving (→ F)

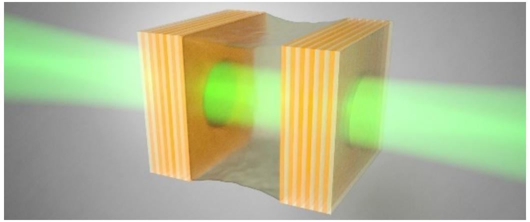
$\Delta = \omega\text{-}\omega_0$ $\quad\quad$ $\Gamma = \kappa_1 + \kappa_2 + \gamma$

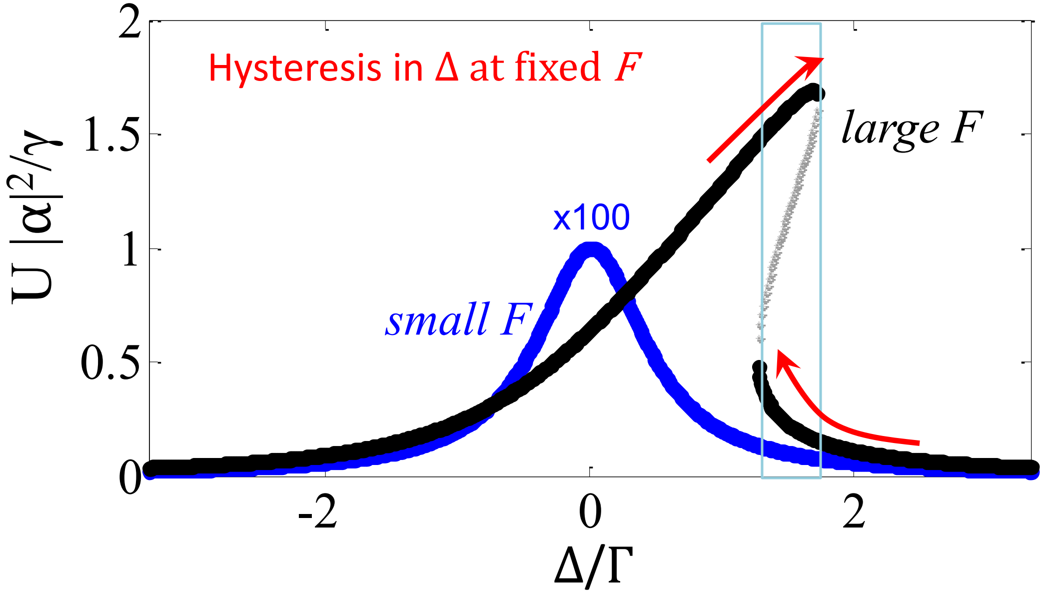


A classical nonlinear cavity

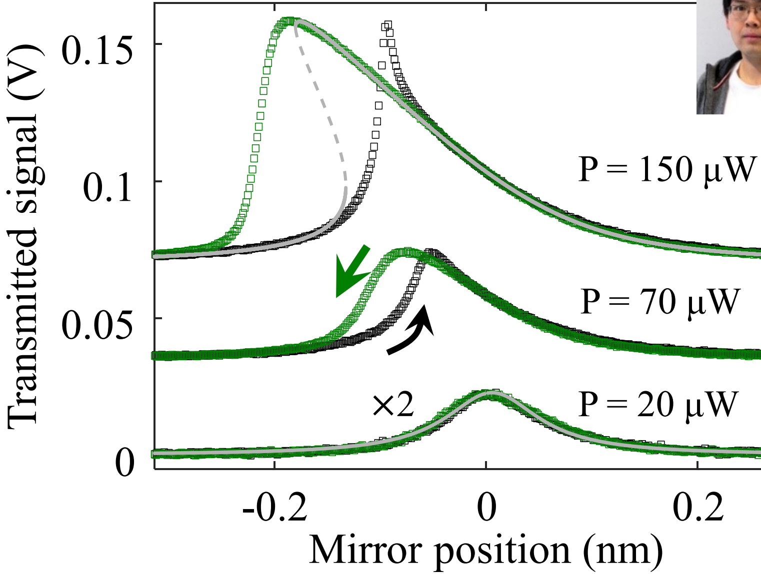
$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \;\; - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F$$

detuning, nonlinearity, loss, driving





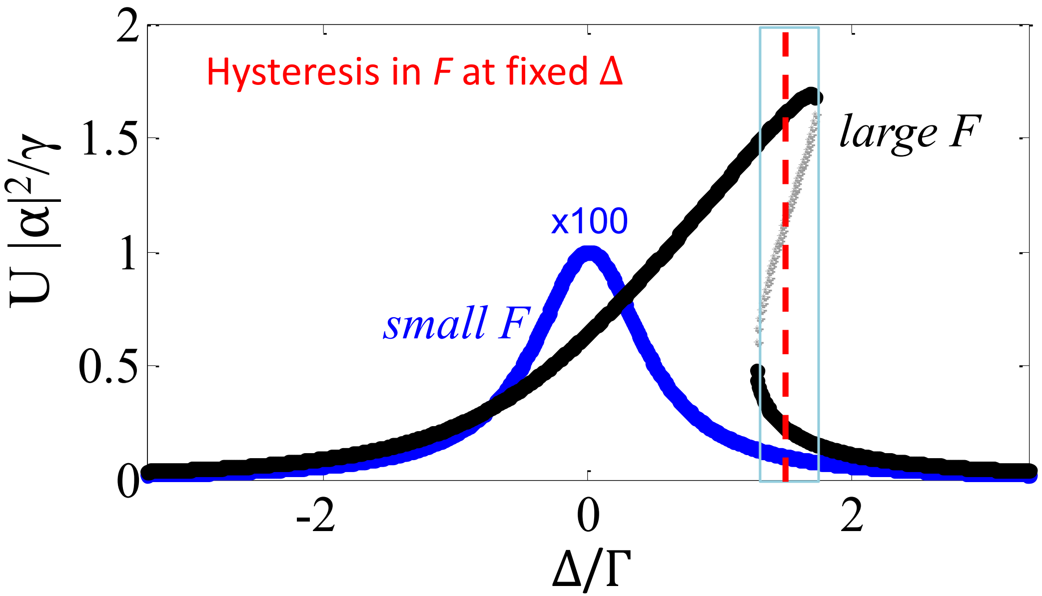
Experiments by Zhou Geng

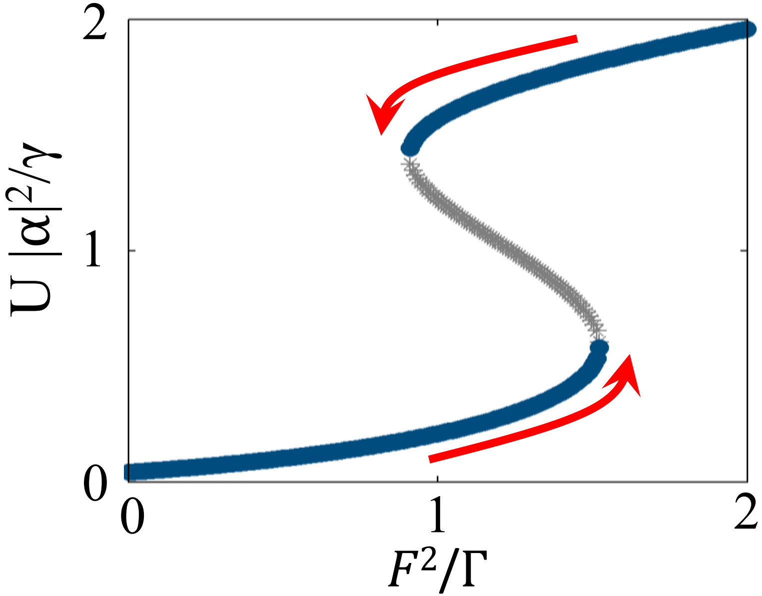


A classical nonlinear cavity

$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \;\; - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F$$

detuning nonlinearity loss driving





A quantum nonlinear cavity

$$\hat{H}(t) = \underbrace{\omega_0 \hat{a}^\dagger \hat{a}}_{} + \overbrace{\frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}}^{\text{interaction}} + \overbrace{\sqrt{I(t)}\left(e^{-i\omega t}\hat{a}^\dagger + e^{i\omega t}\hat{a}\right)}^{\text{driving}}$$

$$\frac{\partial \hat{\rho}(t)}{\partial t} = i\left[\hat{\rho}, \hat{H}(t)\right] + \underbrace{\frac{\gamma}{2}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)}_{\text{loss}}$$

A quantum nonlinear cavity

interaction — driving

$$\hat{H}(t) = \omega_0 \hat{a}^\dagger \hat{a} + \frac{U}{2}\hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a} + \sqrt{I(t)}\left(e^{-i\omega t}\hat{a}^\dagger + e^{i\omega t}\hat{a}\right)$$

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loss

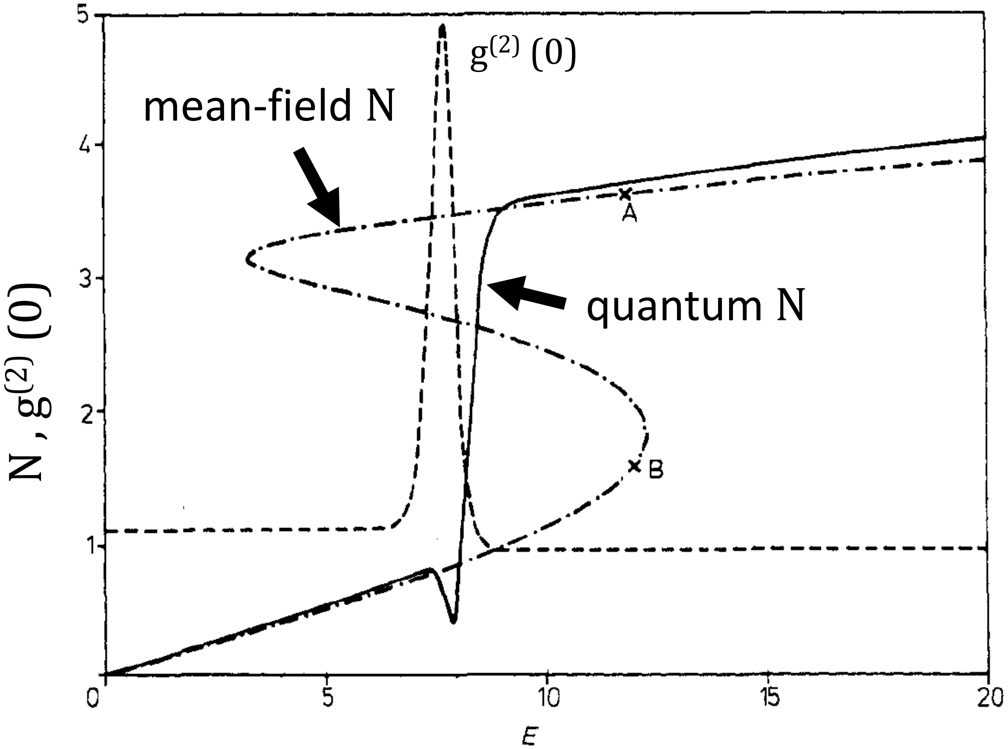
SS density

$$N = \langle \hat{a}^\dagger \hat{a} \rangle = \mathrm{Tr}[\hat{a}^\dagger \hat{a}\hat{\rho}\,]$$

mean-field approx.

$$N = |\alpha|^2$$

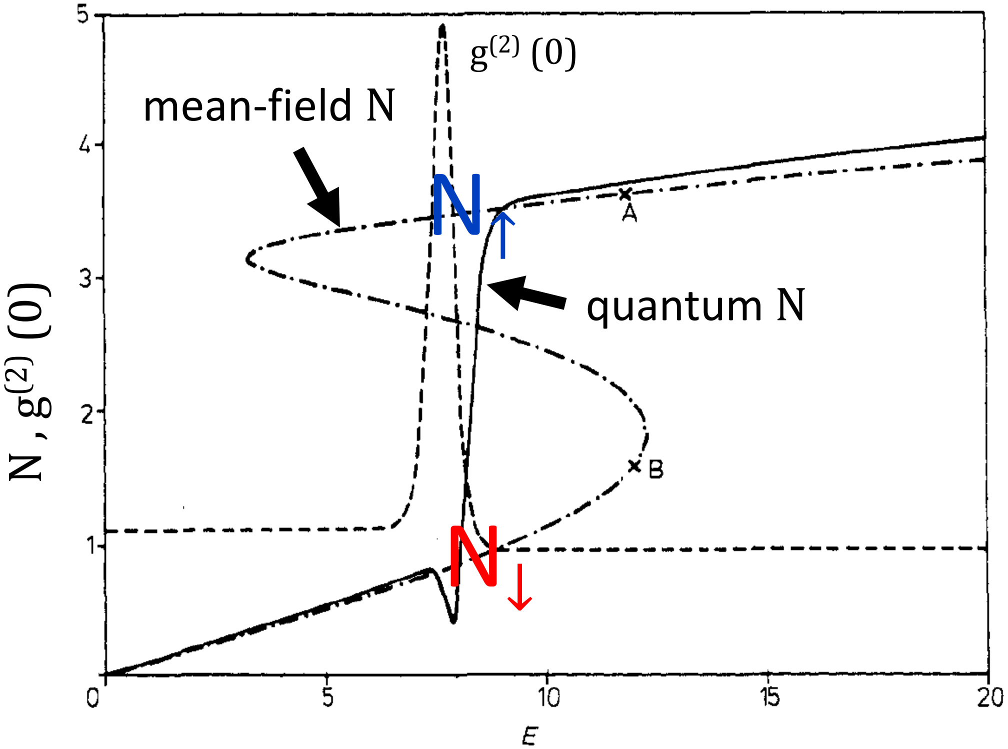
Mean-field bistability vs quantum unique SS



P. D. Drummond & D. F. Walls, J. Phys. A **13**, 725 (1980).

Mean-field bistability vs quantum unique SS

physics of functional complex matter



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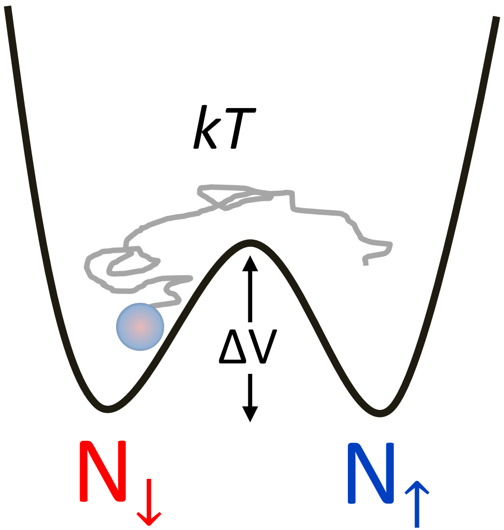
Quantum tunneling in dispersive optical bistability

H. Risken,* C. Savage, F. Haake,† and D. F. Walls
Physics Department, University of Waikato, Hamilton, New Zealand
(Received 23 June 1986)



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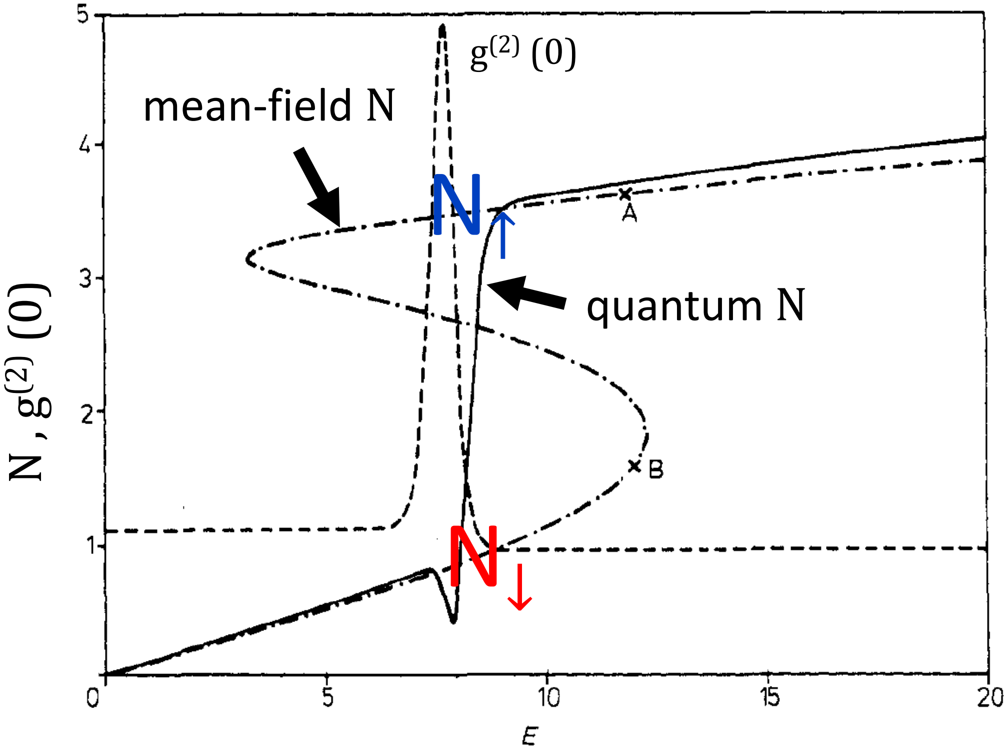

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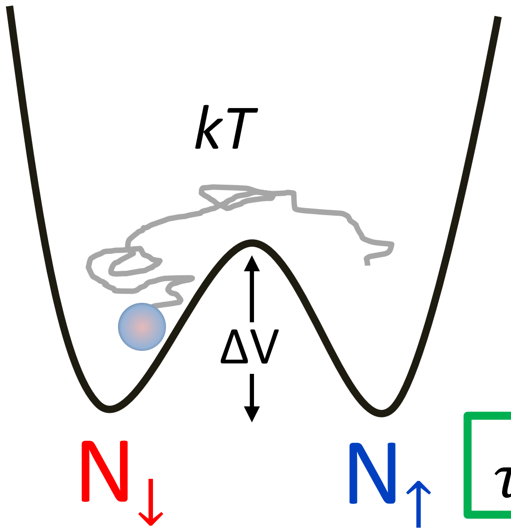

$$\tau \propto e^{\Delta V / kT}$$

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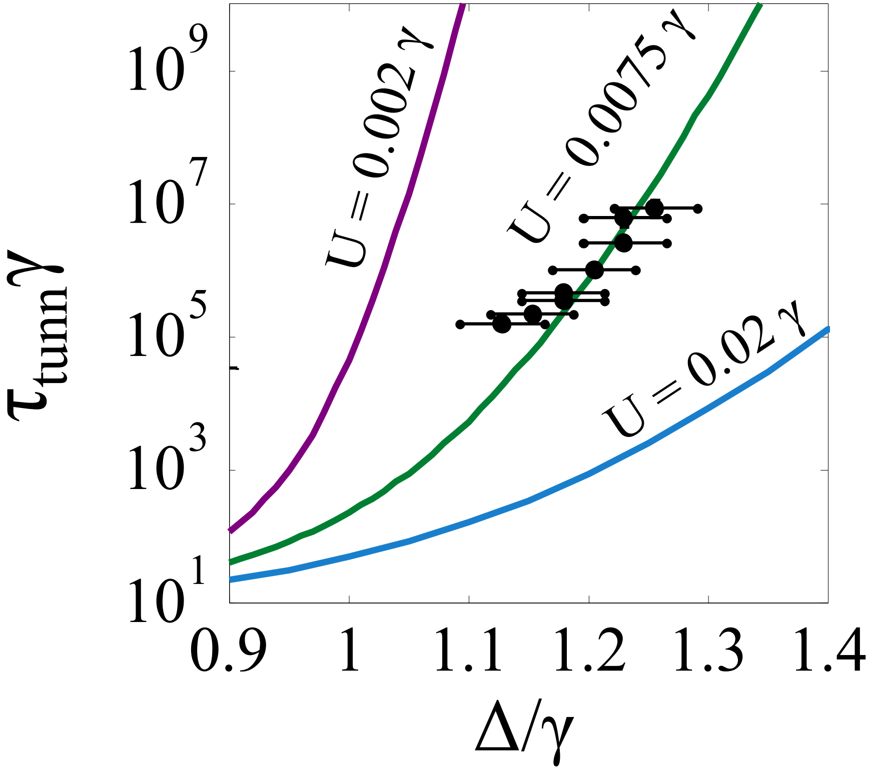
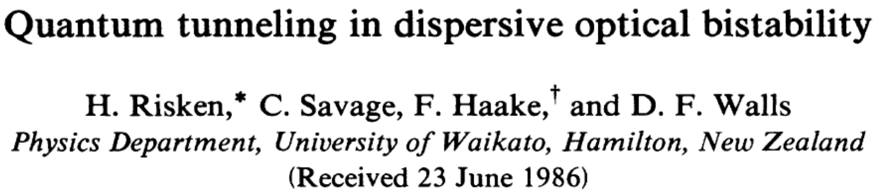
Mean-field bistability vs quantum unique SS

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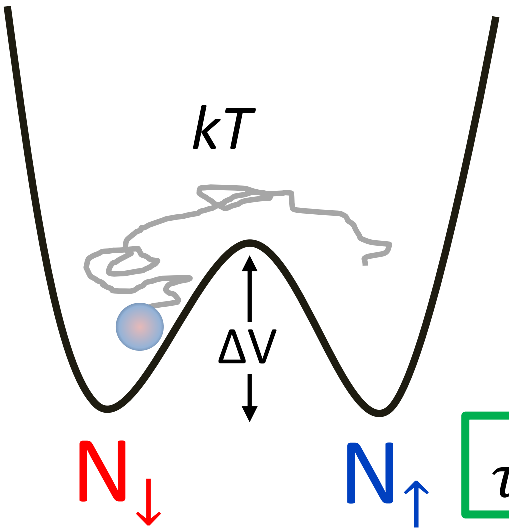

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S.R.K. Rodriguez *et al.*, Phys. Rev. Lett. **118**, 247402 (2017). H. Risken *et al.*, Phys. Rev. A **35**, 1729 (1987).

The influence of noise on a bistable cavity

$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \;\; - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F \;\; + D\tilde{\xi}(t)$$

complex white noise

Valid for $U/\Gamma \lesssim 0.2$

$$D = \sqrt{\frac{\Gamma}{2}}$$

Fluctuation-dissipation

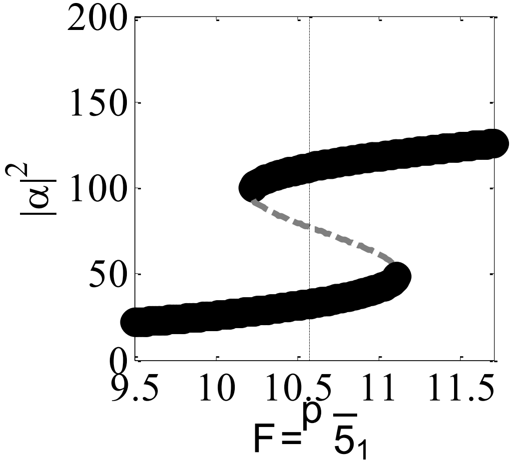
The influence of noise on a bistable cavity

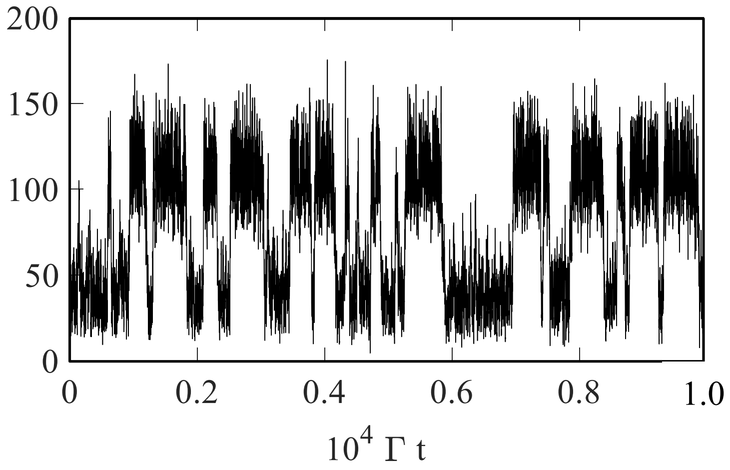
$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \ \ - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F \ + D\tilde{\xi}(t)$$

$$D = \sqrt{\frac{\Gamma}{2}}$$

Fluctuation-dissipation

Δ=1.1Γ





The influence of noise on a bistable cavity

AMOLF physics of functional complex matter

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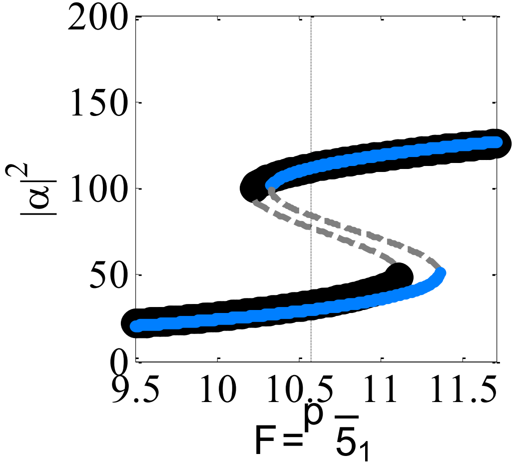
$$D = \sqrt{\frac{\Gamma}{2}}$$

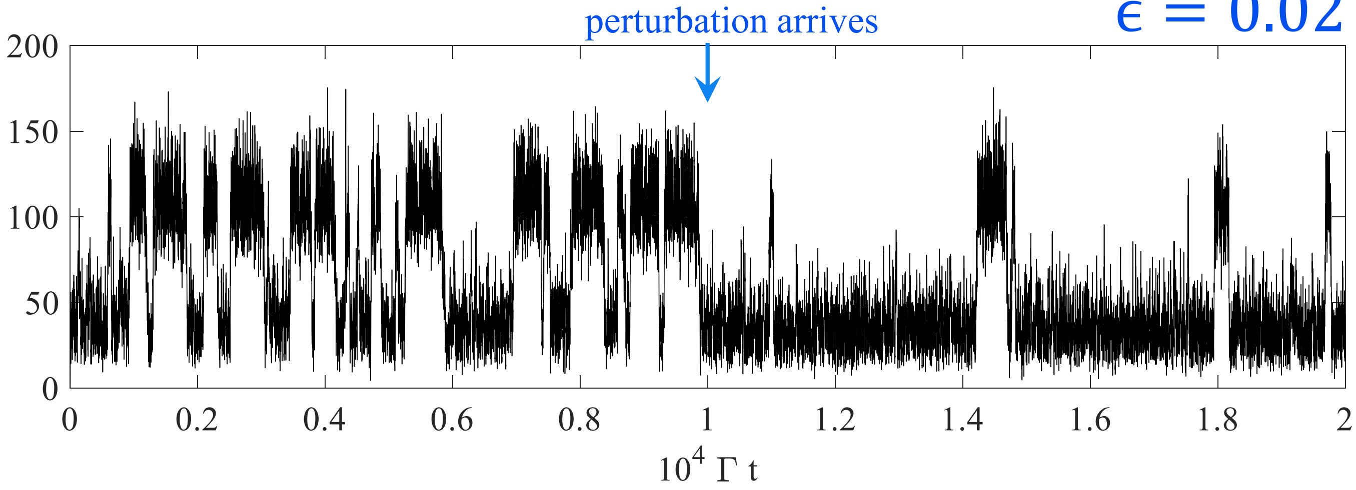
Fluctuation-dissipation

$$\frac{\Delta}{\Gamma} \rightarrow \frac{\Delta}{\Gamma}(1+\epsilon)$$

Δ=1.1Γ

$\epsilon = 0.02$



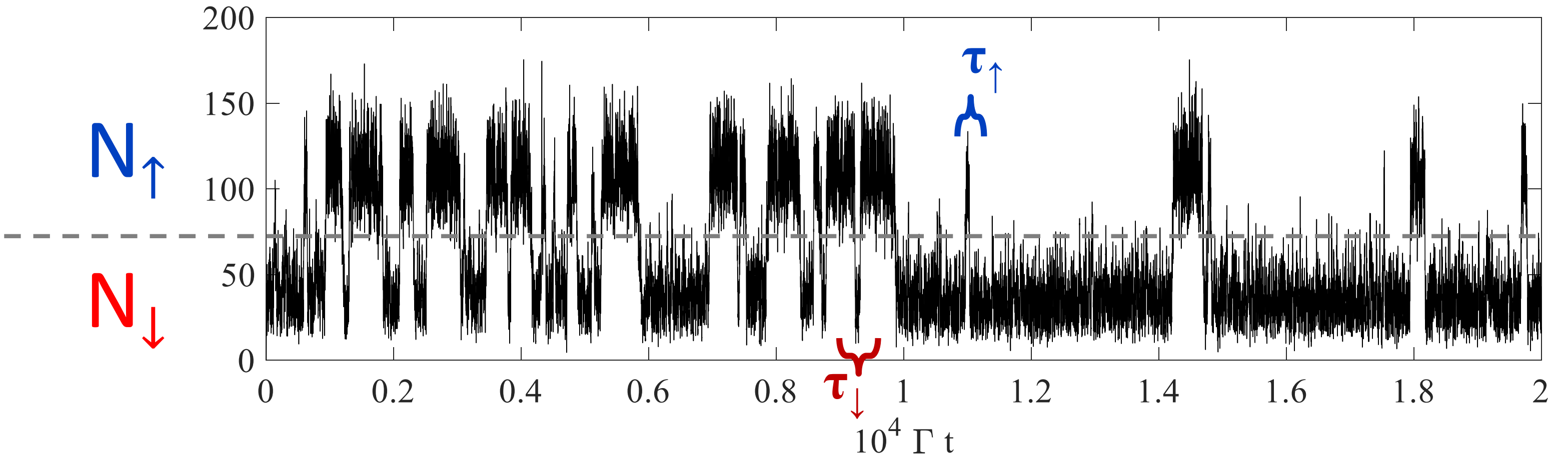


Sensing based on residence time difference

AMOLF physics of functional complex matter

$$i\dot{\alpha} = \left(-\Delta + U(|\alpha|^2 \; - 1) - i\frac{\Gamma}{2}\right)\alpha + i\sqrt{\kappa_1}F \; + D\tilde{\xi}(t)$$

Mean residence time difference: $\delta\tau = \tau_{\uparrow} - \tau_{\downarrow}$

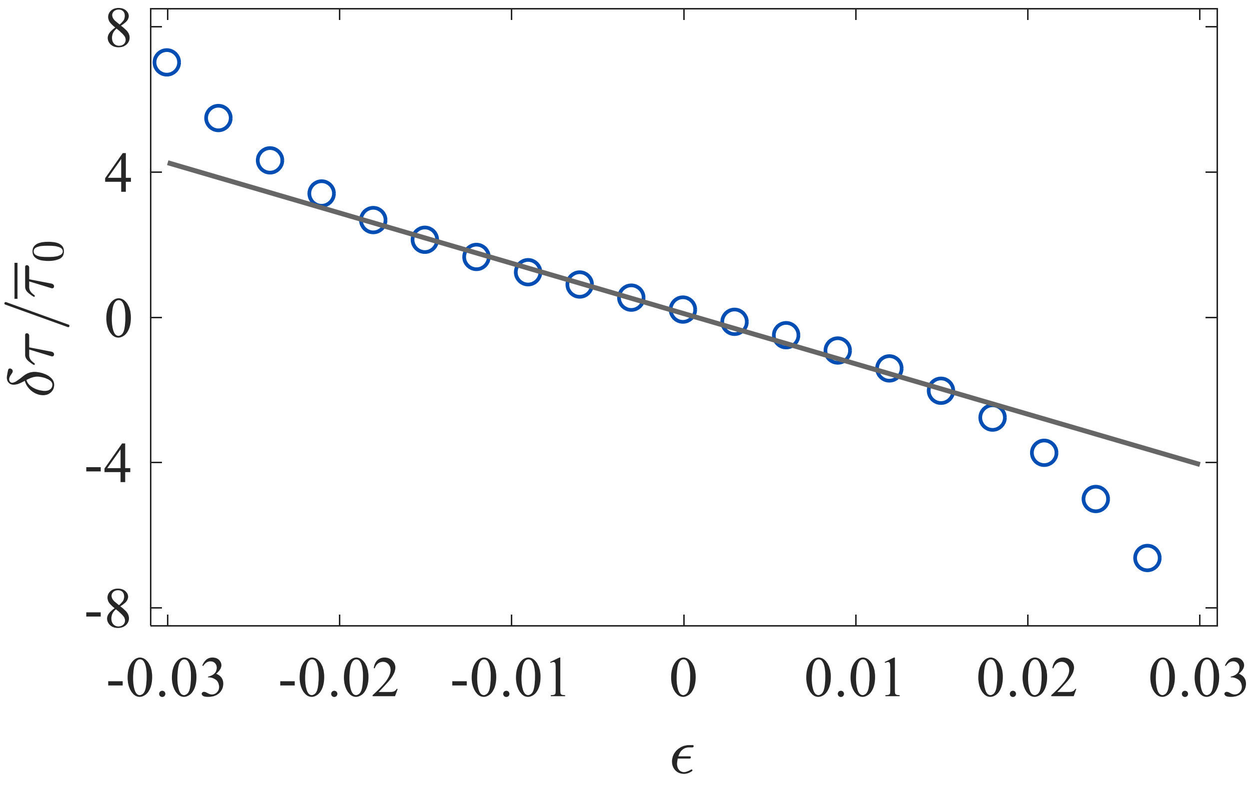


Sensitivity

$$\delta\tau = \tau_{\uparrow} - \tau_{\downarrow}$$

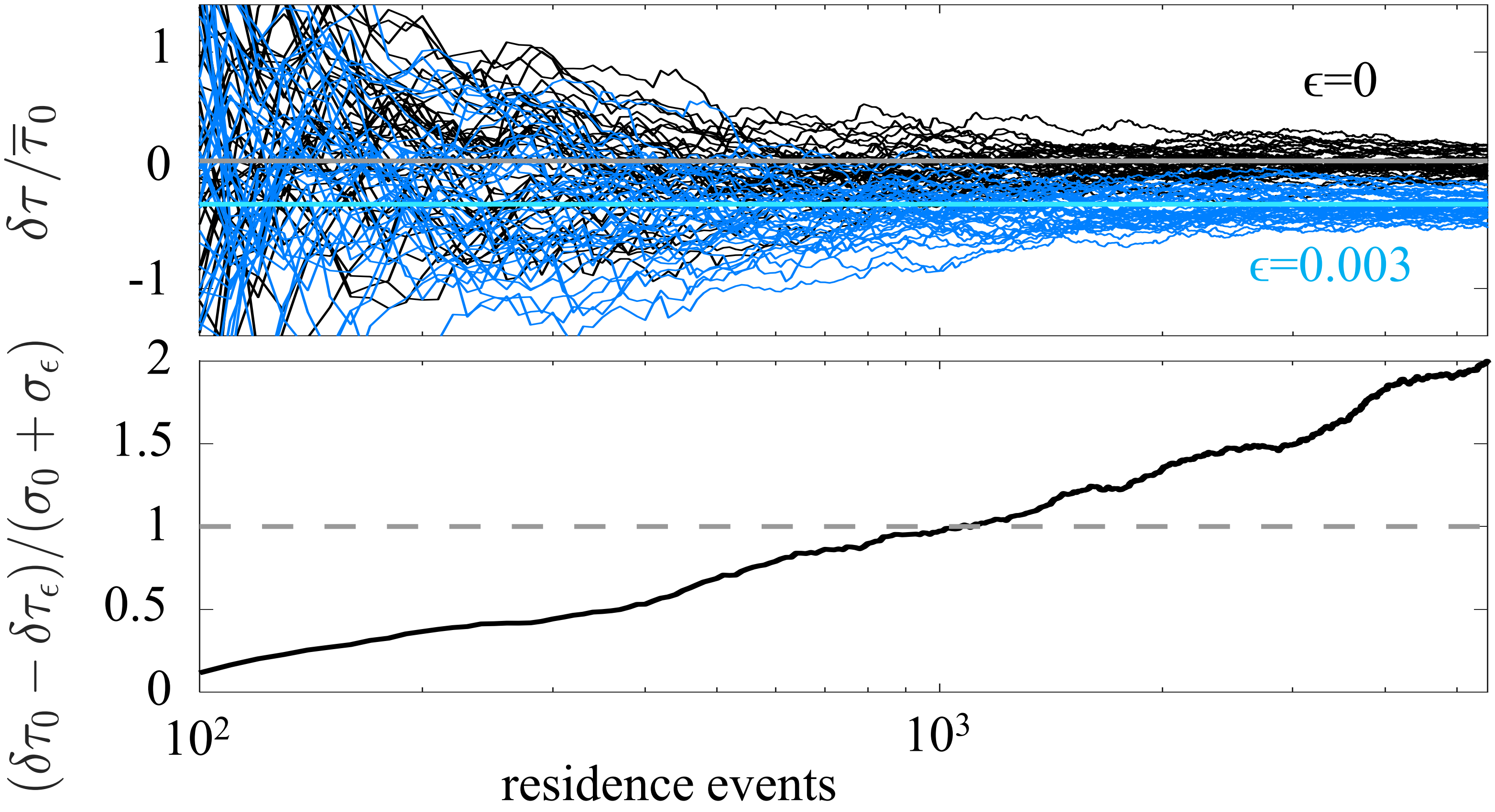
$$\bar{\tau}_0 = \frac{\tau_{\uparrow}(\epsilon = 0) + \tau_{\downarrow}(\epsilon = 0)}{2}$$

$$S = \frac{\partial \delta\tau}{\partial \epsilon} = 139 \text{ s}$$



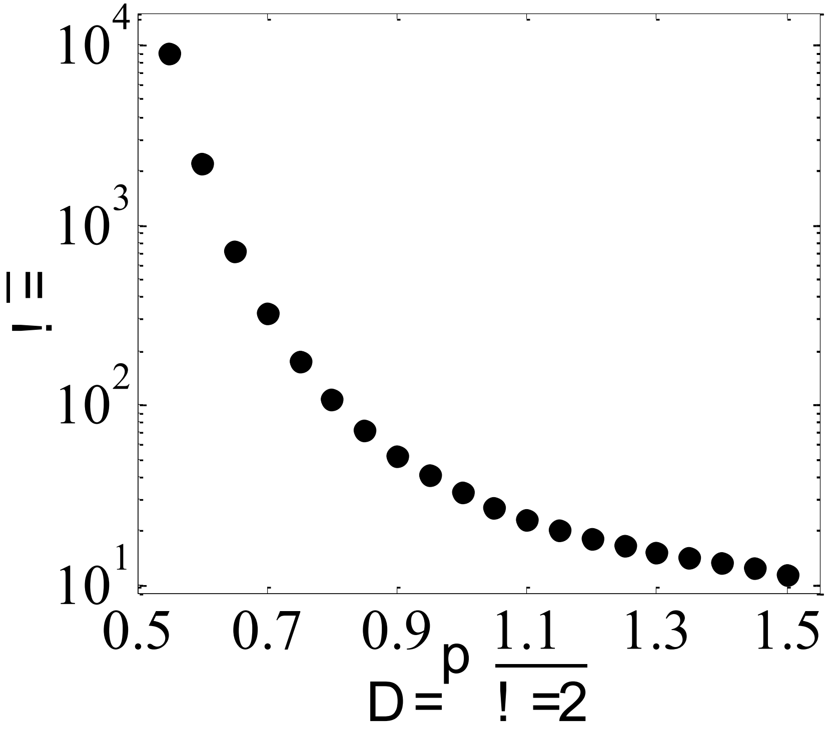
Based on 4000 residence events and 100 noise realizations

How much time we need to measure to detect a certain perturbation?

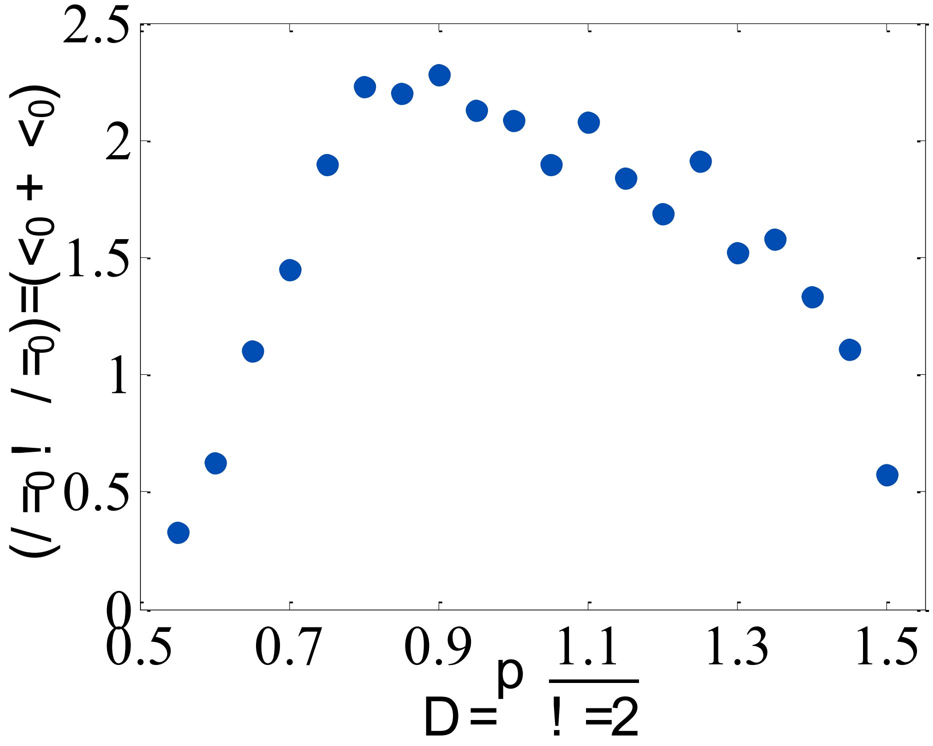


Detection speed & sensitivity

Detection speed increases with noise strength



Maximum sensitivity for finite noise strength



Thank you!

AMOLF physics of functional complex matter

MARIE CURIE ACTIONS

NWO

Veni, Vidi, Vici

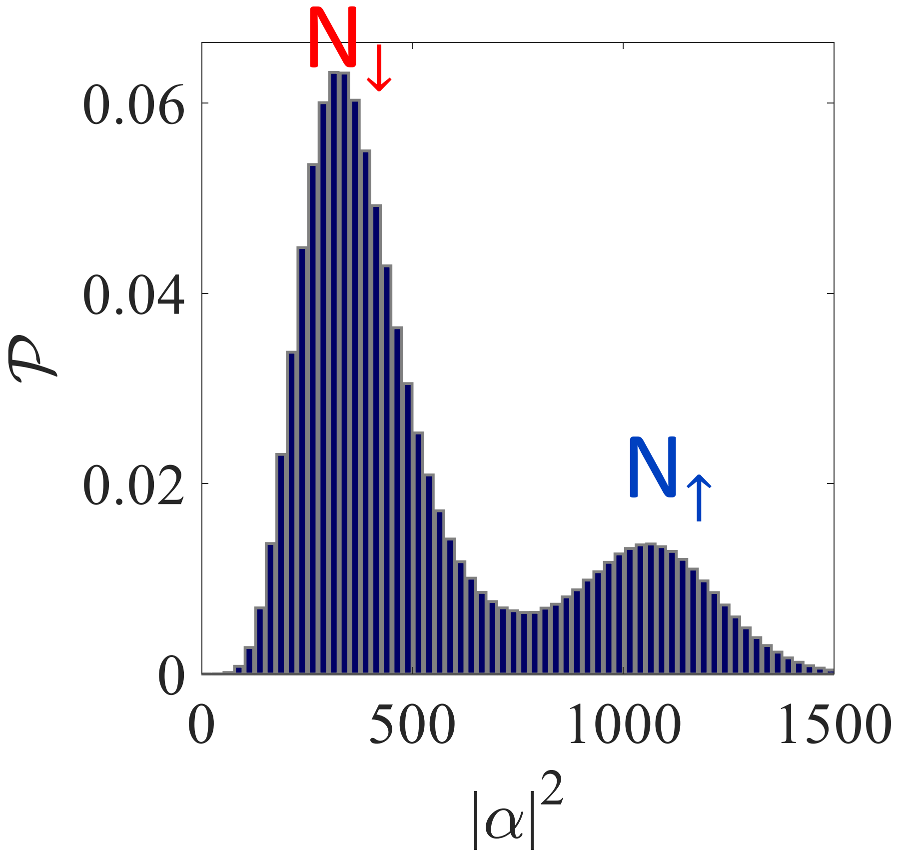
arXiv:1908.05521

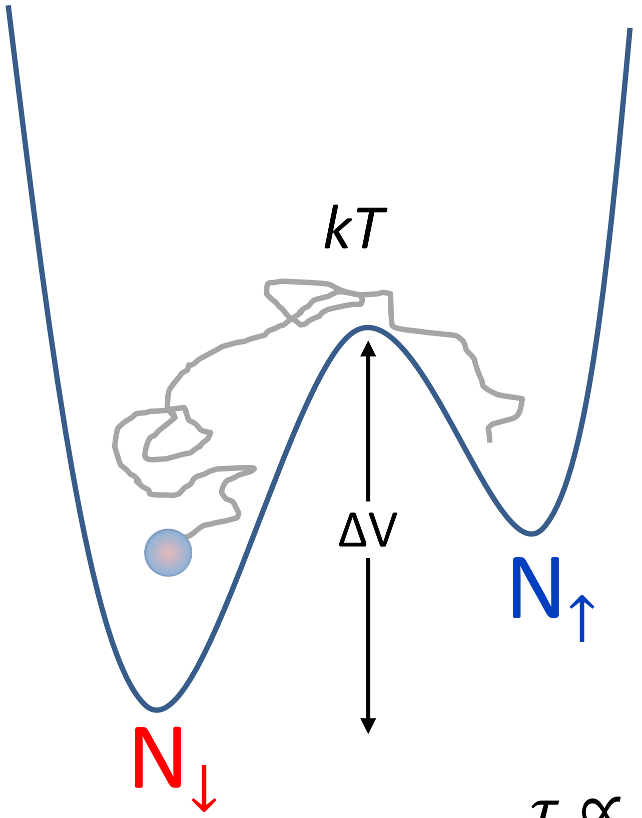
www.amolf.nl

www.srkrodriguez.eu

Back-up slides

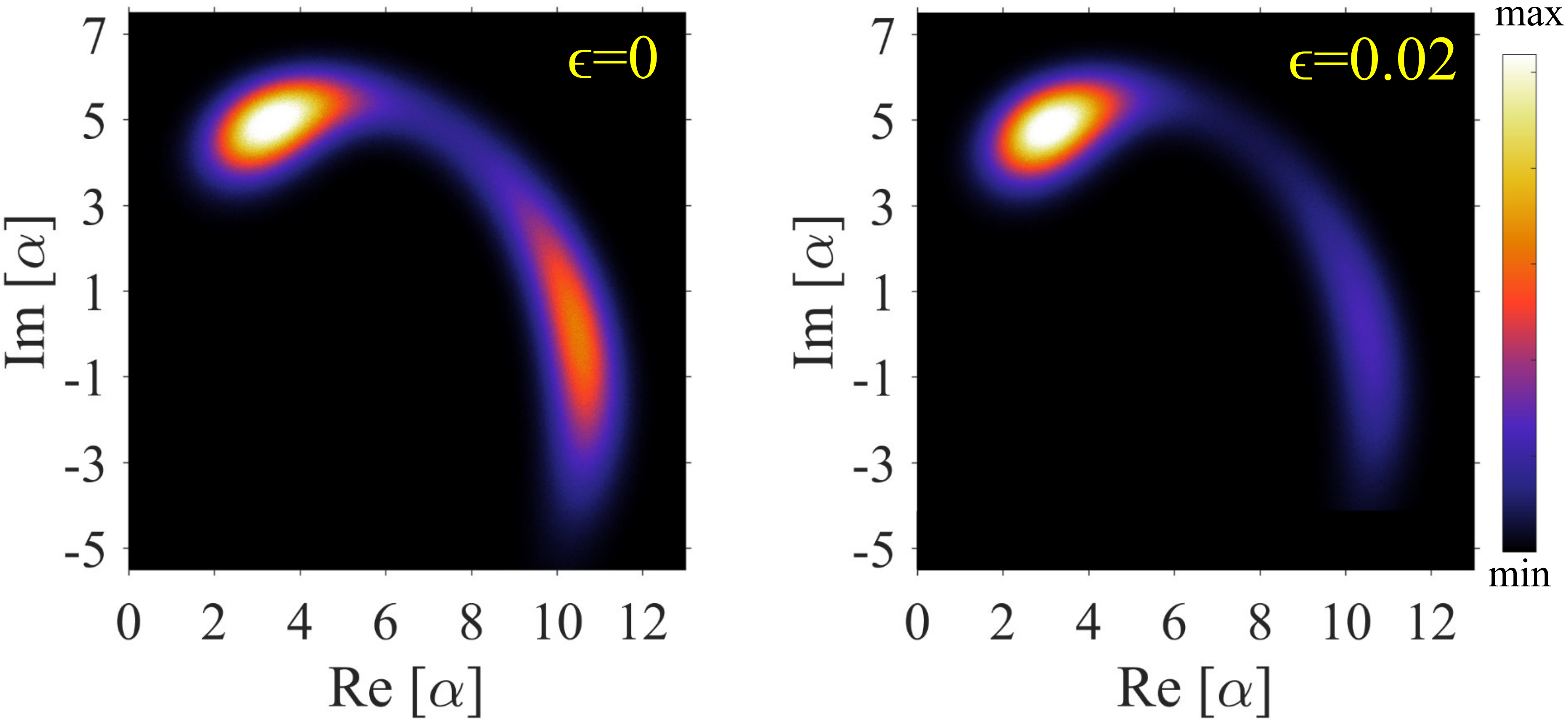
Analogy with a Brownian particle in a DWP



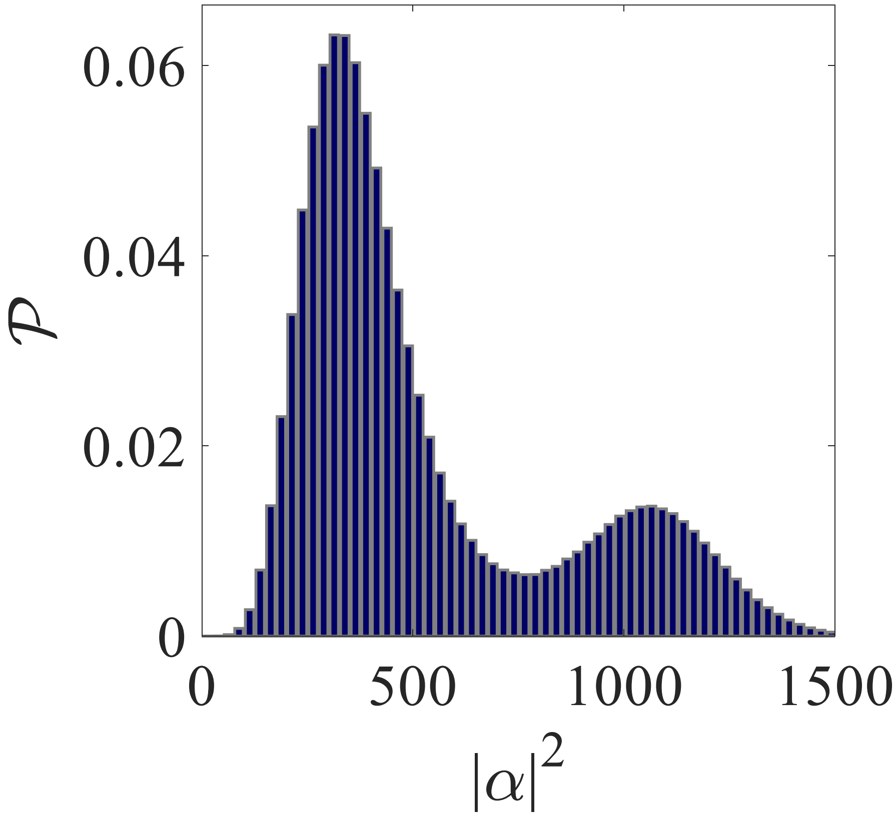


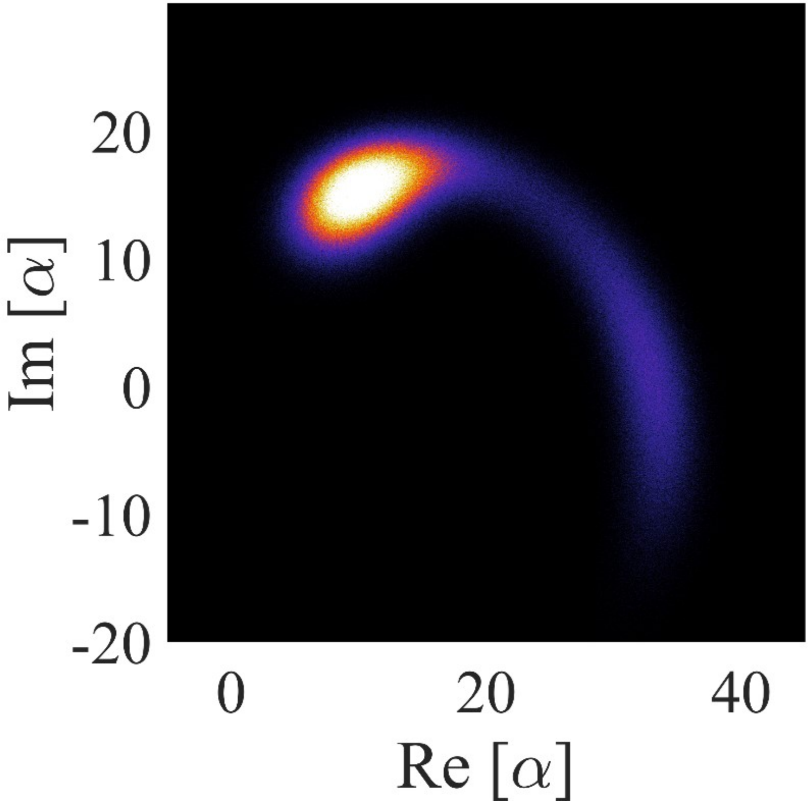
$$\tau \propto e^{\Delta V / kT}$$

A perturbation tilts the effective DWP

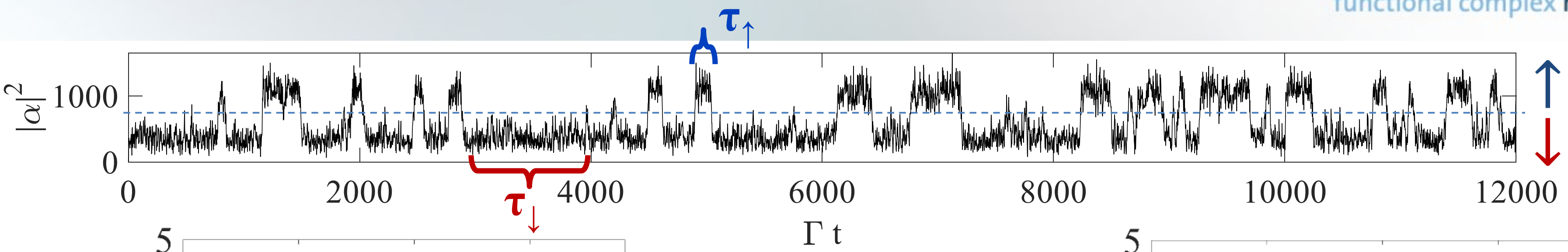


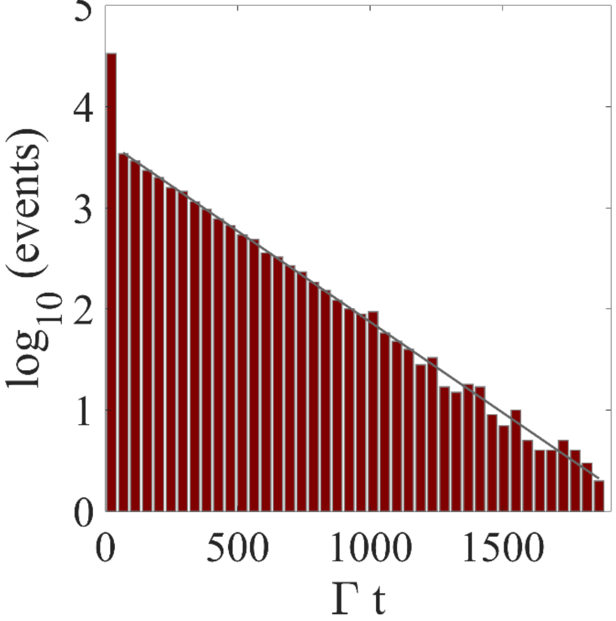
Probability densities





Residence time statistics





Mean residence time difference:

$$\delta\tau = \tau_{\uparrow} - \tau_{\downarrow}$$